IJCIM

International Journal of Complementary and Internal Medicine

RESEARCH ARTICLE

The New Interpretation of Quantum Mechanical Probability and Renormalization Using 4 Mathematical Laws and 5 Physical Laws

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Received: April 17, 2023 **Published:** April 27, 2023

Citation: Rhee H. The New Interpretation of Quantum Mechanical Probability and Renormalization Using 4 Mathematical Laws and 5 Physical Laws. Int J Complement Intern Med. 2023;4(1):156–166. DOI: 10.58349/IJCIM.1.4.2023.00124

Abstract

This article provides new ways to analyze atomic world using Einstein's Relativity's shortest path theory. It defines wave function and the complex number function values with deviation from the shortest path defined in General Relativity. Especially it assigns pure imaginary part to the deviation. Then it defines the new concept so called weighted path by applying 4 mathematical laws and derives probability density of wave function by weighted path concept. Then it introduces 5 physical laws and by applying this new concept of these laws and new wave function concept, it explains in the new way why probability phenomenon occurs, and renormalization concept.

Keywords: physically continuous, The 4 Mathematical Laws, laws of existence, 5 physical laws, microscopic, macroscopic, Hamiltonian Picture, Lagrnagian Picture, Status Functional

1. 4 Mathematical Laws

According to Rhee, assume that a certain physical quantity is assigned to objects. then the 4 rules which will be applied to the quantity is described as follows:

1. If the 'change' or 'difference' of physical quantity while the change of spacetime is infinitesimal is the most symmetrical over all the other changes, then real infinitesimal number ϵ will be assigned to this change. If the change is in the time forward direction, then the number ϵ must be positive number, and if it is in the time backward direction, negative real number must be assigned to the change.

2. If the 'change' or 'difference' of physical quantity while the change of spacetime is infinitesimal is the change which is not allowed by Einstein's Relativity (the most non-symmetrical), then the pure imaginary number $i\epsilon$ will be assigned to this change. This change is spacelike movement of certain particle, or the deviation of the object's path from the General Relativity's shortest path, for example. Assume there are 2 status of an object, and each is described by 2 numbers of physical quantities each. Then 2 following rules must be applied to these 2 numbers.

3. The Rule of Addition: If the physical status (represented by complex number C) consists of 2 distinguishable status which is represented by the complex number A, B, then C is obtained by Addition of A and B. The word distinguishable status means that there is no uncertainty between A and B when they form physical status C.

4. The Rule of Multiplication: If the physical status (represented by complex number C) consists of 2 indistinguishable status which is represented by the complex number A, B, then C is obtained by multiplication of A and B. The word indistinguishable status means that there is non-zero uncertainty between A and B when they form physical status C.

2. Basic Principle

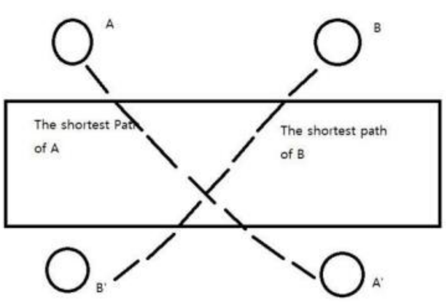

Figure 1. The 2 Identical Infinitesimal Electrons Traveling and Meeting Each Other Inside the Shielded Black Box

Assume there are 2 objects in Fig 1, and the 2 objects are indistinguishable to each other. According to general relativity by Einstein, every object is assigned to travel on the shortest path in spacetime. Each of 2 object a, b is assigned AA' and BB' as the shortest path of them, respectively. On the way of each one's shortest path, each one meets each other at the same 4 spacetime point named C. Meeting point is shielded by "box", so that no one can see the things inside it.

There are 3 things that can happen to the 2 objects.

Case 1: a, b ignores each other at C
Case 2: a, b swaps their path.
Case 3: a, b each travel to their own new paths.

There is a kind of particle in the universe which is in the indistinguishable status between case 1 or 2 above in this situation, especially if the 2 particles (a, b in this case) are identical to each other. That's because no matter whether they ignore (case 1), or they swap (case 2), nobody knows what happens at C and the result is always the same. This phenomenon will be described by the word "physically continuous". This kind of particle is called fermion. If object a travel on AA' and b on BB' which are the most symmetrical path (case 1), then the positive real number must be assigned, However, if a and b change their path after meeting each other at point C (case 2), then pure imaginary number must be assigned to their path 'status'. Their actual movement in the picture above is that they start with the shortest path and at the point C, they are mixed to form indistinguishable status between object and b. Their final status after certain period of time is that the object a and b hardly distinguishable, but the status of them, whether they belong to case 1, case 2 is distinguishable. Therefore, their final status is $\epsilon + i\epsilon'$ in complex number. (ϵ means the case 1, and $i\epsilon'$ means the case 2)

In this way fermion always deviates from their own spacetime shortest path every time, and their path 'status', which stands for this deviation will be called 'wave function' in this paper. Every object with wave function ψ is combined with the wave function ψ^- which forms indistinguishable status with each other and the combination travels on the shortest path. Their final status after certain period of time must be represented by multiplication of ψ and ψ^-.

In this case, ψ^- is called conjugate wave function of wave function ψ. And $\bar{\psi}_d\psi_d$ value in the equation

$$\bar{\psi}_d\psi_d \times V = \bar{\psi}\psi$$

(V space volume) is called probability density of the wave function. This movement can be simplified by the wave function that starts with ψ and ends up with $\bar{\psi}$ on their path or movement. Assume there are 2 objects in Fig 1, and the 2 objects are distinguishable to each other. When these 2 objects collide at the point C, they ignore each other (case 1), or travel on the new shortest path (case 3) in contrast to the case above. In this case, whether these 2 objects ignore or travel on the new path is easily distinguishable, and one can easily find the proportion of case 1 to case 3, that is called coupling constant. Assume a fermion(electron) is traveling on its spacetime path. The path doesn't have to be limited to the spacetime's shortest path (for easy explanation, only inertial reference frame is assumed). Define δx as the amount of movement in the direction, δy and δz means the direction orthogonal to the direction δx. Also define $\delta\tau$ as the time of observed fermion's reference frame, δt as time of observer's frame

3. Application of the 4 Mathematical Laws to Path of The Fermion

First assign infinitesimal real number to the shortest path of the fermion and call it "path" of fermion. If fermion deviates from the shortest path while traveling, then add pure imaginary infinitesimal number to path of the fermion. The magnitude of pure imaginary number of the path represents the amount of deviation. This complex number is called wave function in this paper. Assume that the fermion travels in the spacetime in the time interval from t_0 to t_n and divide this interval to n sections (δt_1 to δt_n).

The path or wave function of the fermion at time interval δt_m is ψ_m then the path of the fermion in the entire time interval from δt_1 to δt_n can be obtained applying "the rule of Multiplication" in the 4 mathematical laws (each wave function is indistinguishable to each other) which leads to the following term: $\Psi = \psi_1\psi_2 \ldots \psi_n$.

This term can also be represented by the following way:
$\Psi = \psi_1\psi_2 \ldots i \times \delta\psi_{n-1}$

where $\delta\psi_{n-1} = \psi_n - \psi_{n-1}$.

The reason for multiply additional pure imaginary number i is because it means change or "deviation" from the original path.

Now assume there is a system which consists of more than one infinitesimal fermion which located in spacetime in spacelike position to each other, then the path for this whole system can be obtained by applying the rule of addition in the 4 mathematical laws to each wave function (or path) of each fermion at every spacetime positions or integration over whole spacetime. However, before the addition, the wave function must undergo the process called normalization, which means:

$$\left|\int_{x_1}^{x_2}\int_{p_1}^{p_2}\psi_1\,\psi_2\ldots\psi_m dx^4 dp^4\right| = n \times \hbar$$

where
$(x_{10} - x_{20}) \times (p_{10} - p_{20}) = n \times \hbar$ Where x_{10} and p_{10} means each component of spacetime and 4 momentum x_i and p_i respectively. ψ_n is each wave function. Here the integrand like $\psi_1\psi_2 \ldots \psi_m$ in the equation above will be called wave density in this paper. And the path which underwent normalization will be called weighted path.

Probability density of electron is defined to be wave density of electron where the wave density represents shortest path. That's why it must always be real number (refer to 1st law in the 4 mathematical laws.) After applying the rule of addition in the 4 mathematical laws (integration of entire wave density of one electron) to the whole one electron Probability density the entire size of "one electron" can be obtained.

4. Definition of 5 physical laws

To define macroscopic and microscopic fermion, The following 2 laws (laws of existence) must be introduced.

1. The law of wave: Every existence including fermion and boson etc., which has energy of its own must be physically continuous (-ref 1) to each other.

2. The law of particle: The whole integration over probability density of every existence must be integer multiples of $\hbar$ and which means the satisfaction of the following equation –

$$\int_{x_1}^{x_2}\int_{p_1}^{p_2}\bar{\psi_f}\,\psi_f\bar{\psi}\psi dx^4 dp^4 = n \times \hbar$$

Where x_1 and x_2 represents every spacetime components, p_1 and p_2 represents every 4-momentume components. In this case each spacetime and 4 momentum components must satisfy the following condition:

$$n \times \hbar = (x_1 - x_2) \times (p_1 - p_2)$$

where n is integer and $\bar{\psi}\psi$ is probability density and $\bar{\psi_f}\psi_f$ is Fourier Transformed probability density. Especially if $p_1 \sim p_2$ and $E_1 - E_2$ goes to infinity, $t_1 \sim t_2$ and $x_1 - x_2$ goes to infinity then the above equation becomes:

$$\int_{x_1}^{x_2} \bar{\psi}\,\psi dx^3 = n \times \hbar$$

To define the movement of the existence the following 3 laws (laws of movement) must also be introduced.

1. The Law of Energy Conservation
2. The Law of Relativity - The same physical law must be applied to any reference frame.
3. The Law of speed limit - every physical element including fermion, boson, etc. must not exceed speed c, and The Law of momentum conservation (These 2 laws are combined to 1 law).

The condition for satisfaction and violation of the laws of movement in inertial reference frame, energy conservation equation of infinitesimal fermion(electron) is:

$$E^2 = m^2c^4 + p^2c^2$$

1) The condition for satisfaction of The Law of Energy Conservation:

$$\bar{\psi}E\delta t\psi = -i\hbar\bar{\psi}\delta\psi$$

or

$$\bar{\psi}E\delta t\psi = i\hbar\delta\bar{\psi}\psi$$

or

$$\bar{\psi}E\psi = -i\hbar\bar{\psi}\frac{\delta\psi}{\delta t}$$

or

$$\bar{\psi}E\psi = i\hbar\frac{\delta\bar{\psi}}{\delta t}\psi.$$

δt means time interval between the wave function ψ and the corresponding conjugate function ψ^{-} while $\delta\tau \rightarrow 0$ in the reference frame of observer. In these equations Energy E must be constant. E must be constant value and will be called energy of ψ

2) The condition for violation of The Law of Energy Conservation:

These equations mean that energy E must not be constant.

$$\bar{\psi}E\delta t\psi \neq -i\hbar\bar{\psi}\delta\psi$$

or

$$\bar{\psi}E\delta t\psi \neq i\hbar\delta\bar{\psi}\psi$$

or

$$\bar{\psi}E\psi \neq -i\hbar\bar{\psi}\frac{\delta\psi}{\delta t}$$

or

$$\bar{\psi}E\psi \neq i\hbar\frac{\delta\bar{\psi}}{\delta t}\psi$$

2. 1) The condition for satisfaction of The Law of Relativity:

$$\bar{\psi}m\delta t\psi = -i\hbar\bar{\psi}\delta\psi$$

or

$$\bar{\psi}m\delta t\psi = i\hbar\delta\bar{\psi}\psi$$

or

$$\bar{\psi}m\psi = -i\hbar\bar{\psi}\frac{\delta\psi}{\delta t}$$

or

$$\bar{\psi}m\psi = i\hbar\frac{\delta\bar{\psi}}{\delta t}\psi$$

δt means time interval of fermion ψ's reference frame between ψ and corresponding conjugate wave function ψ^{-} while $\delta x \rightarrow 0$. m means rest mass in each inertial reference frame.

2) The condition for violation of The Law of Relativity:

$$\bar{\psi}m\delta\tau\psi \neq -i\hbar\bar{\psi}\delta\psi$$

or

$$\bar{\psi}m\delta\tau\psi \neq i\hbar\delta\bar{\psi}\psi$$

or

$$\bar{\psi}m\psi \neq -i\hbar\bar{\psi}\frac{\delta\psi}{\delta\tau}$$

or

$$\bar{\psi}m\psi \neq i\hbar\frac{\delta\bar{\psi}}{\delta\tau}\psi$$

3. 1) The condition for satisfaction of The Law of speed limit - the Law of momentum conservation:

$$\bar{\psi} p \delta x \psi = -i\hbar \bar{\psi} \delta \psi$$

or

$$\bar{\psi} p \delta x \psi = i\hbar \delta \bar{\psi} \psi$$

or

$$\bar{\psi} p \psi = -i\hbar \bar{\psi} \frac{\delta \psi}{\delta x}$$

or

$$\bar{\psi} p \psi = i\hbar \frac{\delta \bar{\psi}}{\delta x} \psi$$

δx means distance between ψ^{-} and ψ. Here p means momentum in each inertial reference frame.

2) The condition for violation of The Law of speed limit - the Law of momentum conservation:

$$\bar{\psi} p \delta x \psi \neq -i\hbar \bar{\psi} \delta \psi$$

or

$$\bar{\psi} p \delta x \psi \neq i\hbar \delta \bar{\psi} \psi$$

or

$$\bar{\psi} p \psi \neq -i\hbar \bar{\psi} \frac{\delta \psi}{\delta x}$$

or

$$\bar{\psi} p \psi \neq i\hbar \frac{\delta \bar{\psi}}{\delta x} \psi$$

- microscopic fermion: the one defined to be the one which satisfies the law of wave and violates the law of particle.

- macroscopic fermion: the one defined to be the one which satisfies the law of particle and violates the law of wave.

microscopic existence always violates 2 among the 3 laws of movement, while macroscopic existence must always satisfy all the 3 laws.

5. Picture

The concept of Picture is the kind of frame in each of which the microscopical fermion satisfies and violates different laws.

Hamiltonian Picture - It is the picture in which The Law of Relativity is always satisfied and 2 other laws are always violated by microscopical fermion.

Lagrangian Picture - It is the picture in which The Law of Energy Conservation is always satisfied, and 2 other laws are always violated by microscopical fermion.

6. Status Functional

Status Functional B is the functional of which $B \times B^{-}$

is weighted path which magnitude represents the amount of deviation from the shortest path in each picture. (B^{-} conjugate Status Functional with which the equation $B \times B^{-} = 0$ means the shortest path of the fermion(existence))

7. Status Functional and 5 physical laws

I. The Law of Existence

1. The law of wave

- Violation: $B \times B^{-} \neq 0$

- Satisfaction: $B \times B^{-} = 0$

2. The law of particle

- Satisfaction:

$$\int B \times \bar{B_f} dp^4 dx^4 = \int \psi \bar{\psi} dx^3 = n\hbar \quad \text{(n is integer)}$$

- Violation:

$$\int B \times \bar{B_f} dp^4 dx^4$$

is not integer multiples of $\hbar$ and $\int d_4 p d_4 x$ is integration in both 4 momentum and 4 spacetime, B_f^{-} is Fourier Transformed conjugate Status Functional

II. The Law of Movement

1. The Law of Energy Conservation

- Violation

$$\frac{\delta B}{\delta \psi} \neq \frac{\delta\left(\frac{\delta B}{\delta\left(\frac{\delta \psi}{\delta t}\right)}\right)}{\delta t} - \frac{\delta\left(\frac{\delta B}{\delta\left(\frac{\delta \psi}{\delta x}\right)}\right)}{\delta x}$$

where $\delta\tau \rightarrow 0$ and δt and δx is the corresponding spacetime quantity in the inertial reference frame.

- Satisfaction

$$\frac{\delta B}{\delta\psi} = \frac{\delta\left(\frac{\delta B}{\delta\left(\frac{\delta\psi}{\delta t}\right)}\right)}{\delta t} - \frac{\delta\left(\frac{\delta B}{\delta\left(\frac{\delta\psi}{\delta x}\right)}\right)}{\delta x}$$

2. The Law of Relativity

- Violation

$$\frac{\delta B}{\delta\psi} \neq \frac{\delta\left(\frac{\delta B}{\delta\left(\frac{\delta\psi}{\delta t}\right)}\right)}{\delta t}$$

where $\delta x \to 0$

- Satisfaction

$$\frac{\delta B}{\delta\psi} = \frac{\delta\left(\frac{\delta B}{\delta\left(\frac{\delta\psi}{\delta t}\right)}\right)}{\delta t}$$

3. The Law of Speed Limit and Momentum Conservation

- Violation

$$\frac{\delta B}{\delta\psi} \neq \frac{\delta\left(\frac{\delta B}{\delta\left(\frac{\delta\psi}{\delta x}\right)}\right)}{\delta x}$$

where $\delta t \to 0$

- Satisfaction

$$\frac{\delta B}{\delta\psi} = \frac{\delta\left(\frac{\delta B}{\delta\left(\frac{\delta\psi}{\delta x}\right)}\right)}{\delta x}$$

* Hamiltonian Functional - is status functional of fermion(electron) in Hamiltonian Picture

* Lagrangian Functional - is status functional of fermion(electron) in Lagrangian Picture

* Momentum Functional - is status functional of fermion(electron) in both pictures.

8. Momentum Functional P

The functional P originally has following form:

$$P = -i\hbar\bar{\psi} \times \frac{\delta\psi}{\delta x}$$

$$\bar{P} = i\hbar\psi \times \frac{\delta\bar{\psi}}{\delta x}$$

If the fermion microscopically violates The Law of Speed Limit and Momentum Conservation, to represent macroscopical fermion, macroscopical compensation term must be added to only P, and that leads to :

$$P_m = -i\hbar\bar{\psi} \times \frac{\delta\psi}{\delta x} + CP$$

Where CP means macroscopical compensation and according to The Law of Particle,

$$\int P_m \times \bar{P}_f dp^4 dx^4 = n\hbar$$

$$\int CP dp^4 dx^4$$

Assume that n and h is 1

represents macroscopic compensation and satisfies following inequality:

$$0 \leq \int CP dp^4 dx^4 < \frac{1}{2}$$

Therefore,

$$\frac{1}{2} < \int P \times \bar{P}_f dp^4 dx^4 = \nabla x \nabla p \leq 1$$

where ∇x and ∇p means standard deviation from the mean value of distance (x) and momentum (p), respectively. This is same as Heisenberg's inequality.

9. Hamiltonian density Functional H

Hamiltonian density Functional H is status functional of fermion(electron) in Hamiltonian Picture. Hamiltonian Picture is the picture in which The Law of Relativity is always satisfied, and 2 other laws of movement are always violated by microscopical fermion.

To satisfy this conditions, Hamiltonian density Functional H must be a kind of Status Functional and satisfies the following condition:

$$\frac{\delta H}{\delta \psi} = \frac{\delta\left(\frac{\delta H}{\delta\left(\frac{\delta \psi}{\delta t}\right)}\right)}{\delta t}$$

Therefore, Hamiltonian density Functional H must be in the form of

$$H = -i\hbar\bar{\psi}\frac{\delta\psi}{\delta t}$$

Furthermore, to satisfy The law of wave, Hamiltonian density Functional H must satisfy the following additional condition:

$$H = \bar{\psi}\left(\frac{\hbar^2\nabla^2}{2m}\right)\psi$$

,

$$\bar{H} = \bar{\psi}m\psi$$

so that

$$H\bar{H} = \bar{\psi}\left(\frac{\hbar^2\nabla^2}{2m}\right)\psi \times \bar{\psi}m\psi \sim \bar{\psi}p^2\psi \neq 0$$

because

$$\bar{\psi}p^2\psi$$

is zero if the wave travels in the gravitational shortest path in inertial reference frame and $\delta x \rightarrow 0$. Therefore, According to Wiki (2)

$$H = -i\hbar\bar{\psi}\frac{\delta\psi}{\delta t} = \bar{\psi}E\psi = \bar{\psi}\left(\frac{\hbar^2\nabla^2}{2m} - V\right)\psi$$

where ∇ means positional variation of ψ and V means macroscopic compensation of the deviation from the shortest path. In the equation (2), the term

$$\bar{\psi}\left(\frac{\hbar^2\nabla^2}{2m}\right)\psi$$

means microscopic deviation of fermion wave function ψ from the fermion's free wave shortest path, and the term $\bar{\psi}V\psi$ means the macroscopic deviation. The fermion's microscopic mean movement can be defined by the following formula:

$$\bar{\psi}E\delta t\psi dxdydz = \bar{\psi}\left(\frac{\hbar^2\nabla^2}{2m} - V\right)\psi\delta t dxdydz = \bar{\psi} \times 1 \times \hbar \times \psi$$

Ideally, even though microscopic deviation is allowed, macroscopic deviation must be zero, therefore,

$$\int_1 \bar{\psi}E\delta t\psi dxdydz = \int_1 \bar{\psi}\left(\frac{\hbar^2\nabla^2}{2m}\right)\psi\delta t dxdydz = 1 \times \hbar(constant)$$

($\int_1$ means the whole part of physically continuous one electron wave function.) and,

$$\int_1 \bar{\psi}V\delta t\psi dxdydz = 0$$

However, In reality there is also the macroscopic deviation and the allowed macroscopic deviation can be expressed by following inequalities:

$$\int_1 \bar{\psi}\left(\frac{\hbar^2\nabla^2}{2m}\right)\psi\delta t dxdydz > \frac{1}{2} \times \hbar$$

$$\int_1 \bar{\psi}V\delta t\psi dxdydz < \frac{1}{2} \times \hbar$$

If there is also compensating electric potential in the universe, one can assume that the violation of The Law of Energy Conservation, The Law of Speed Limit - Momentum Conservation must be compensated by exchanging energy and momentum between electric potential and fermion(electron) wave function. Then the fermion inside these compensating electric potential travels on the new shortest path defined inside the potential field.

For easy explanation, assume that the fermion(electron) is the one which surrounds and circulates around the Hydrogen proton, so that the compensating electric potential is defined as the electric potential around the Hydrogen proton.

* Eigenfunction: Here eigenfunction is the electron wave function which follows the new shortest path inside the hydrogenic potential while circulating around hydrogen proton exactly only once.

$$\int_1 \bar{\psi_n}\left(\left(\frac{\hbar^2}{2m}\nabla^2\right)-V_H\right)\psi_n dxdydzdt = \int \bar{\psi_n}E_n\psi_n dxdydzdt$$

where the symbol $\int_1$ means the integration of whole electron in the time interval of exactly one circulation and ψn eigenfunction, and E_n corresponding energy of the eigenfunction, V_H is the hydrogenic potential. Corresponding energy E_n of eigenfunction in this case (around hydrogen proton) is limited to only several discrete values, and the integer n in the expression E_n only means the order of such values. One can define new fermion wave function ψ_a which satisfies following equation:

$$\bar{\psi_a}\delta E\psi_a = \bar{\psi_a}\left(\frac{\hbar^2\nabla_2^2}{2m}-V_2\right)\psi_a$$

where δE means non-conservation energy difference of the fermion wave function ψ, ∇_2, and V_2 means microscopic and macroscopic deviation of the fermion of the shortest path inside electric potential, respectively. Like ψ it also satisfies following equation and inequalities:

(5)

$$\bar{\psi_a}|\delta E\delta t|\psi_a dxdydz = \bar{\psi_a}\left(\frac{\hbar^2\nabla_2^2}{2m}-V_2\right)\delta t\psi_a dxdydz = \bar{\psi}\times 1\times\hbar\times\psi$$

and

(6)

$$\int_1 \bar{\psi_a}\left(\frac{\hbar^2\nabla_2^2}{2m}\right)\delta t\psi_a dxdydz > \frac{1}{2}\times\hbar$$

(7)

$$\int_1 \bar{\psi_a}V_2\delta t\psi_a dxdydz < \frac{1}{2}\times\hbar$$

The inequality (6) means microscopic deviation of fermion ψ_a from the path ψ and the inequality (7) means macroscopic deviation of fermion ψ_a from the path ψ.

The fermion ψ_a must also be compensated by additional electric potential corresponding to V_2 and travel on new shortest path inside hydrogenic potential added by the above field.

There is also the mixed field of ψ and ψ_a which satisfies the following condition:

$$\bar{\psi}'E'\psi' = \bar{\psi}'\left(\frac{\hbar^2\nabla^2}{2m}-V'\right)\psi'$$

where ψ' is the mixed fermion wave, and

$$E' = E + \delta E$$

$$V'$$

means hydrogenic potential plus the additional. The solution for this equation must be:

$\psi' = \sum a_n\psi_n$

where ψ_n is so called eigenfunction, ψ' means mean wave function of electron which have circulated around Hydrogen proton N times.

a_n is complex number, and real part of it means non-exchanged part of ψ_n with the other eigenfunction, according to (ref-1), and imaginary part of it means, on the other hand, exchanged part with the other eigenfunction, therefore, deviated part of ψ_n

Assume that the shortest path is defined as the path decided by fermion inside only hydrogenic potential excluding additional field.

Quantum mechanical measurement of the energy in this case is defined as measurement done to fermion which circulated the Hydrogen proton only once, and deviation part ψ_a

($|\int_1\bar{\psi}\delta E\delta t\psi dxdydz|$) goes to zero in the case.

Then $\psi = \psi_n$

where n is one of integer. It means the electron Quantum mechanically measured stays exactly at eigenfunction(eigenstate) at every circulation. In contrast

If ($|\int_1\bar{\psi}\delta E\delta t\psi dxdydz|$) is not zero, and if electron of wave function ψ have circulated around Hydrogen proton N($N >> 1$)times, then

$$\psi = \sum a_n\psi_n$$

In every circulation, as long as

$$|\delta E\times\delta t| < 1/2\times\hbar, \delta E = E_n - E$$

the electron can stay at any eigenstate n. It means that the eigenstate of electron must be chosen randomly among n eigenstate. In contrast, after N circulation,

$$\psi = \sum a_n\psi_n$$

Therefore, although the eigenstates are randomly chosen, the probablility, of "choice" must be $|a_n|$

10. Lagrangian Function L for fermion

Lagrangian density Functional L is status functional of fermion(electron) in Lagrangian Picture. Lagrangian Picture is the picture in which The Law of Energy Conservation is always satisfied, and 2 other laws of movement are always violated by microscopical fermion. Therefore, according to Wiki (3)

$$\frac{\delta L}{\delta \psi}=\frac{\delta\left(\frac{\delta L}{\delta\left(\frac{\delta \psi}{\delta t}\right)}\right)}{\delta t}-\frac{\delta\left(\frac{\delta L}{\delta\left(\frac{\delta \psi}{\delta x}\right)}\right)}{\delta x} \quad (8)$$

and $LL^{-} \neq 0$.

The Lagrangian that satisfies (8) and $LL^{-} \neq 0$ is:

$$L_b=\bar{\psi}(i\gamma^{\mu}\delta_{\mu}-m)\psi$$

There must be a macroscopic compensation for L_b, and it must be added to L_b because it is always clearly distinguished from L_b. Addition of macroscopic compensation for the deviation of this L_b leads to the following Lagrangian density Functional L is, according to Wiki (4):

$$L=\bar{\psi}(i\gamma^{\mu}\delta_{\mu}-m)\psi-e\bar{\psi}\gamma^{\mu}A_{\mu}\psi$$

Lagrangian functional and Feynman Diagram

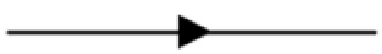

Figure 2. Feynman Diagram representing fermion (electron) only.

The weighted path of whole macroscopic electron of the Feynman Diagram above can be calculated using the condition for satisfaction of The Law of Particle:

$$\int L_b \times L_{bf}^{-} dp^4 dx^4=\int \psi\bar{\psi}dx^3$$

(Here, $\hbar = 1$ and L_{bf}^{-} is the Fourier transformed functional of L_b^{-}) It is equal to 1 if macroscopic term is ignored and The Lagrangian functional L_b and L_{bf}^{-} must be replaced by $\psi^{-}\psi$ and $\psi^{-}{}_f\psi_f$

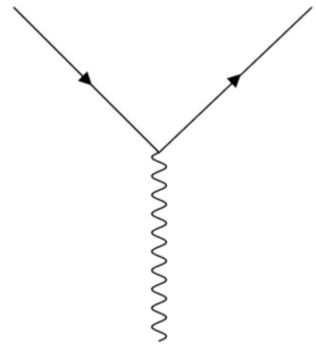

Figure 3. Feynman Diagram representing fermion(electron) and external electric field. Scattering Amplitude of the Feynman Diagram above can be calculated using the condition for satisfaction of The Law of Particle with the macroscopic part added to only L_b, not to L_b^{-} :

$$\int L \times L_{bf}^{-} dp^4 dx^4=1$$

The Scattering Amplitude of the Feynman Diagram above is the value of the 2nd term of the following integration:

$$\int_{x_1}^{x_2}\int_{p_1}^{p_2}(\bar{\psi_f}\psi_f\bar{\psi}\psi+\bar{\psi_f}\psi_f e\bar{\psi}\gamma^{\mu}A_{\mu}\psi\bar{\psi}\psi)dp^4dx^4=1$$

Here

$$(x_{2n}-x_{1n})(p_{2n}-p_{1n})=1$$

(x_{2n} and x_{1n}, p_{2n} and p_{1n} represents each component of spacetime x_1, x_2 and 4-momentum p_1, p_2, respectively). $\int_{x1}^{x2}$ represents 4 dimension integration and x_1 and x_2 represents 4 components spacetime. The same thing applies to $\int_{p1}^{p2}$ and p_1 and p_2 In the equation above,

$$\int_{x_1}^{x_2}\int_{p_1}^{p_2}\bar{\psi_f}\psi_f\bar{\psi}\psi dp^4dx^4>\frac{1}{2}$$

because ψ_f and ψ in $\psi_f^{-}\psi_f\ \psi^{-}\psi$ represents microscopic deviation of fermion and

$$\int_{x_1}^{x_2}\int_{p_1}^{p_2}\bar{\psi_f}\psi_f e\bar{\psi}\gamma^{\mu}A_{\mu}\psi\bar{\psi}\psi<\frac{1}{2}$$

because it represents macroscopic deviation (compensation).

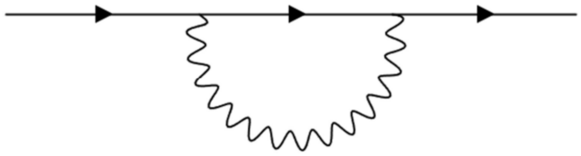

Figure 4. Feynman Diagram representing fermion (electron) with internal loop. The weighted path of the Feynman Diagram above can be calculated by multiplying the following term to the case of Figure 2:

$$\int L \times \bar{L_f} dp^4 dx^4 \sim \int_{x_1}^{x_2} \int_{p_1}^{p_2} e_b^2 \bar{\psi} \gamma^\mu A_\mu \psi \bar{\psi_f} \gamma^\mu A_{\mu f} \psi_f dp^4 dx^4$$

(The other terms in the integration $\int L \times L_f^- dp^4 dx^4$ is ignored) In this case, $\psi \psi_f^-$ in the middle of integrand must be replaced by 1, $A_\mu A_{\mu f}$ must be replaced by $G_p G_x$ where G_p is momentum space photon propagator and G_x is position space photon propagator. Here, G_p and G_x must be, according to Wiki (5):

$$G_p = \frac{-i g_{\mu\nu}}{k^2}$$

$$G_x = R_p \theta(x_1, x_2)$$

where θ (x_1, x_2) is step function which has the value 1 between x_1 and x_2 and 0 elsewhere, and R_p is the renormalization factor. ψ at the beginning of the integrand and ψ^- at the end must be combined and replaced by $H_p H_x$ where H_p is momentum space fermion propagator and H_x is position space fermion propagator which is the following (according to Wiki (5)):

$$H_p = \frac{i(/p - /k + m)}{(p-k)^2 - m^2}$$

$$H_x = R_f \theta(x_1, x_2)$$

where $\theta(x_1, x_2)$ is step function which has the value 1 between x_1 and x_2 and 0 elsewhere, and R_f is the renormalization factor.

$$(x_{2n} - x_{1n})(p_{2n} - p_{1n}) = 1$$

(x_{2n} and x_{1n}, p_{2n} and p_{1n} represents each component of spacetime x_1, x_2 and 4- momentum p_1, p_2, respectively.)

The term renormalization in this paper is defined to decide coupling constant e by the following way:

$$e^2 = e_b^2 \times R_f \times R_p$$

so that R_f and R_p is removed in the above integration.

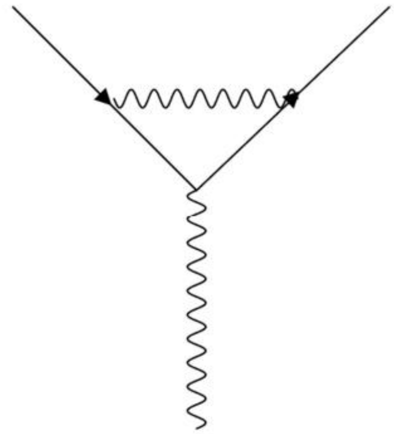

Figure 5. Feynman Diagram representing fermion (electron) which interact with external electric potential.

Scattering Amplitude of the Feynman Diagram above can be calculated by multiplying the following term to the case of Figure 3:

$$\int L \times \bar{L_f} dp^4 dx^4 \sim \int_{x_1}^{x_2} \int_{p_1}^{p_2} e_b^2 \bar{\psi} \gamma^\mu A_\mu \psi \bar{\psi_f} \gamma^\mu A_{\mu f} \psi_f dp^4 dx^4 = 1$$

Acknowledgement

None

Conflicts of Interest

None

Funding

None

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